Elasticity

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- **Elasticity**:- The property of a body, by virtue of which body regains its original size and shape when the applied force is removed, is known as **elasticity** and the body is known as **Elastic body**.

  Eg. Spring, rubber, skin, etc.

- **Plasticity**:- The property of a body, by virtue of which body does not regain its original size and shape when the applied force is removed, is known as **plasticity** and the body is known as **Plastic body**.

  Eg. Plastic paper, clay, putty, etc.

- **Rigidity:-** The property of a body, by virtue of which body does not change its original size and shape when the force is applied is known as **Rigidity.**

  Eg. Wall, Black board, duster, etc.

Stress:- The restoring force per unit area is known as stress.
If **F** is the force applied and **A** is the area of cross section of the body.

Stress = F/A

The SI unit of stress is N/m^2.

Strain:- It is defined as change in dimensions per unit original dimensions.

Strain= change in dimensions/original dimensions

Strain has no unit.

There are 3 types of stress:-

1) Tensile or Longitudinal Stress:-

If the applied force produces change in length of a body, the stress associated is called as Tensile Stress.

Longitudinal stress = $F/A = Mg/\prod r^2$

2) Volume stress :-

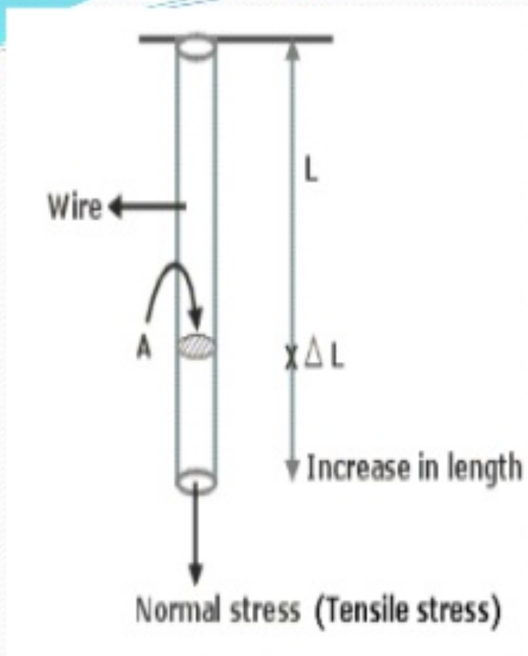
If the applied force produces change in volume of a body, the stress associated is called as Volume Stress.

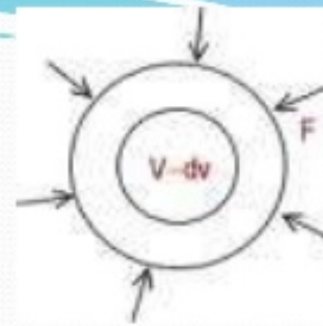
Volume Stress = $A.dP/A = dP$

3) Shear stress :-

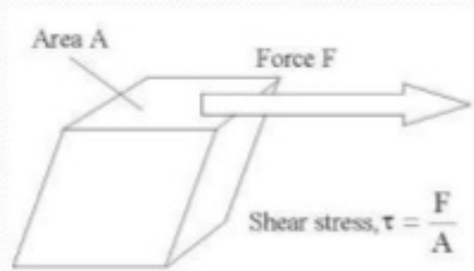
If the applied force produces change in shape of a body, the stress associated is called as Shear Stress.

Shear Stress = Tangential force/ Area





Volume stress



Longitudinal strain

Shear stress

There are 3 types of strain:-

1) Tensile or Longitudinal Strain:-

The change in the length per unit original length of the body is known as longitudinal strain.

Longitudinal strain = l/L

2) Volume stress :-

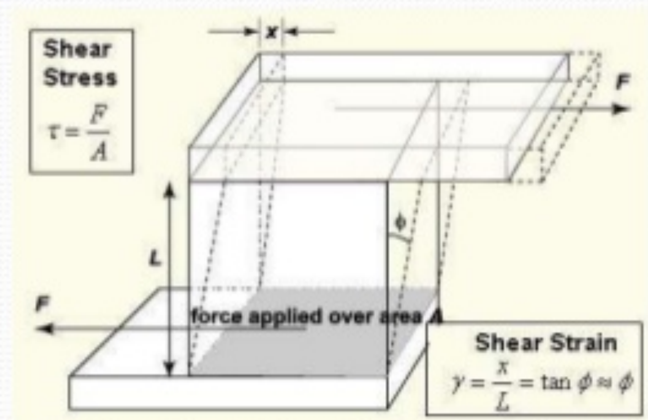
The change in the volume per unit original length of the body is known as volume strain.

Volume Strain = dV/V

3) Shear stress :-

The ratio of relative displacement of any layer to its perpendicular distance from fixed surface is known as shear strain.

Shear Strain = X/L



HOOKE'S LAW:-

Statement:- "*Within elastic limit, stress is directly proportional to strain.*"

Thus,

$$\text{stress} \propto \text{strain}$$

$$\text{stress} = M \times \text{strain}$$

where M = proportionality constant called as **modulus of elasticity.**

Therefore, M= Stress/strain

There are 3 types of elastic constants:-

1) Young's Modulus (Y)
2) Bulk Modulus (K)
3) Modulus of Rigidity(η)

1) Young's Modulus (Y):-

It is the ratio of longitudinal stress to the longitudinal strain.

$$Y = \text{Longitudinal stress} / \text{Longitudinal strain}$$

$$= \frac{(Mg/\pi r^2)}{(l/L)}$$

$$= \frac{MgL}{\pi r^2 l}$$

S.I unit of Y is **N/m^2**

Dimensions are **$[L^{-1} M^1 T^{-2}]$**

2) Bulk Modulus(K):-

It is the ratio of volume stress to the volume strain.

$$K = \frac{\text{volume stress}}{\text{volume strain}}$$

$$= \frac{dP}{(dV/V)}$$

$$= \frac{V.dP}{dV}$$

S.I unit of Y is **N/m^2**

Dimensions are **$[L^{-1} M^1 T^{-2}]$**

3) Modulus of Rigidity(η):-

It is the ratio of shearing stress to the shearing strain.

η = shearing stress/shearing strain.

= (F/A)/θ

= F/A θ

Poisson's Ratio(σ):

"It is defined as the ratio of lateral strain to the longitudinal strain."

σ = lateral strain/longitudinal strain

but, lateral strain= Change in dimension/original dimension

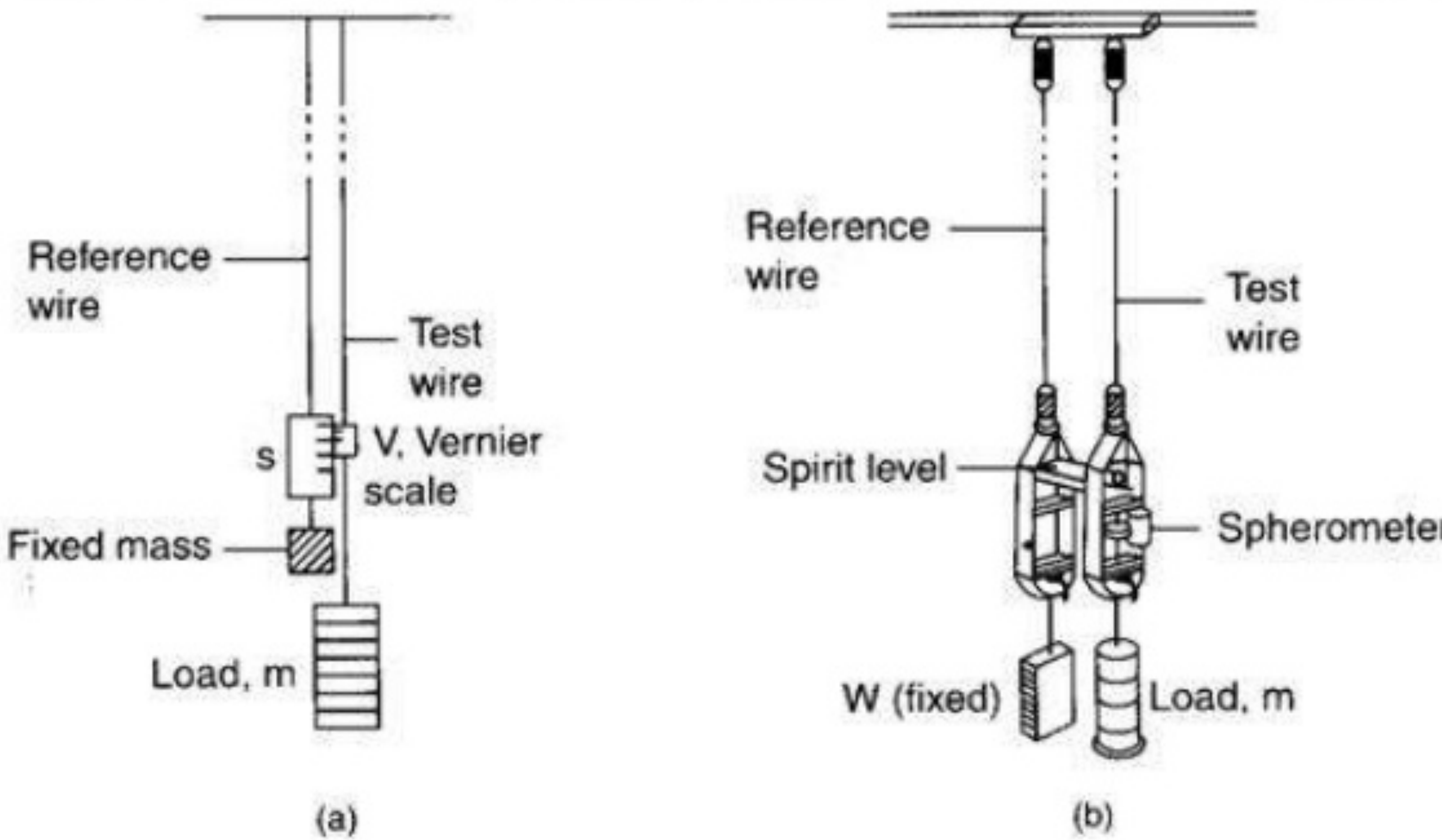
and longitudinal strain = Change in length/original length

$$\therefore \sigma = \frac{(-dW/W)}{(l/L)}$$

$$= \frac{dW.L}{Wl}$$

Negative sign indicates that increase in length is accompanied by decrease in its transverse dimensions.

Determination of Young's Modulus of the Material of a Wire

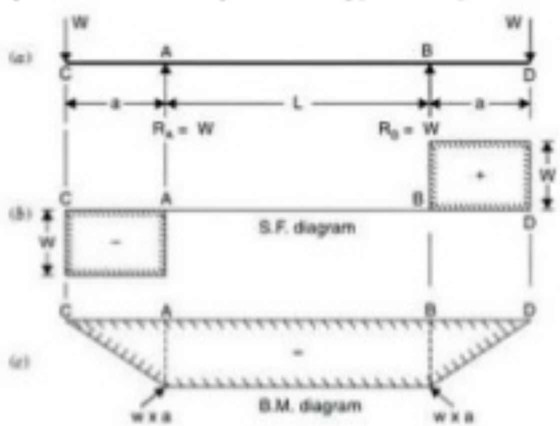


4.1 SIMPLE BENDING OR PURE BENDING

- When some external force acts on a beam, the shear force and bending moments are set up at all the sections of the beam
- Due to shear force and bending moment, the beam undergoes deformation. The material of the beam offers resistance to deformation
- Stresses introduced by bending moment are known as bending stresses
- Bending stresses are indirect normal stresses

4.1 SIMPLE BENDING OR PURE BENDING

- When a length of a beam is subjected to zero shear force and constant bending moment, then that length of beam is subjected to pure bending or simple pending.
- The stress set up in that length of the beam due ... simple bending stresses



4.1 SIMPLE BENDING OR PURE BENDING

- Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the over hanging portion
- In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
- Hence the portion AB is said to be subjected to pure bending or simple bending

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The material of the beam is isotropic and homogeneous. Ie of same density and elastic properties throughout
- The beam is initially straight and unstressed and all the longitudinal filaments bend into a circular arc with a common centre of curvature
- The elastic limit is nowhere exceeded during the bending
- Young's modulus for the material is the same in tension and compression

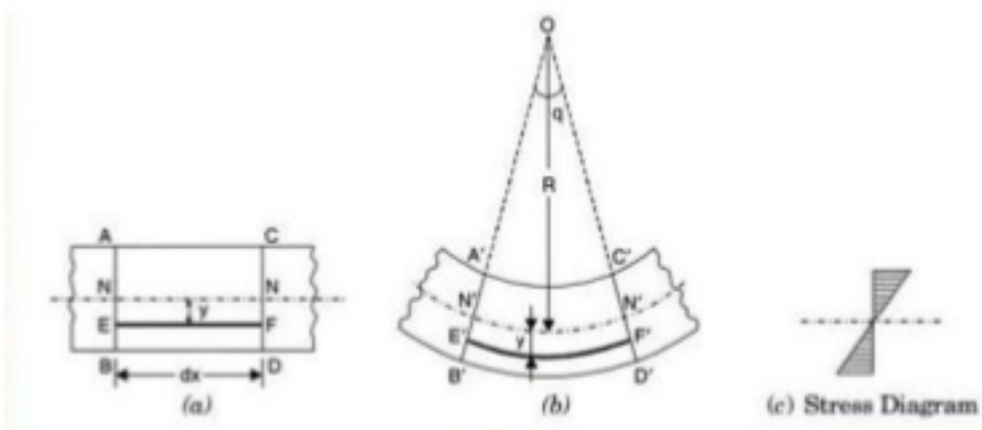
4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The transverse sections which were plane before bending remain plane after bending also
- Radius of curvature is large compared to the dimensions of the cross section of the beam
- There is no resultant force perpendicular to any cross section
- All the layers of the beam are free to elongate and contract, independently of the layer, above or below it.

4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Consider a layer EF at a distance y from the neutral axis. After bending this layer will be deformed to E'F'.
- Strain developed= (E'F'-EF)/EF

EF=NN=dx=R x θ

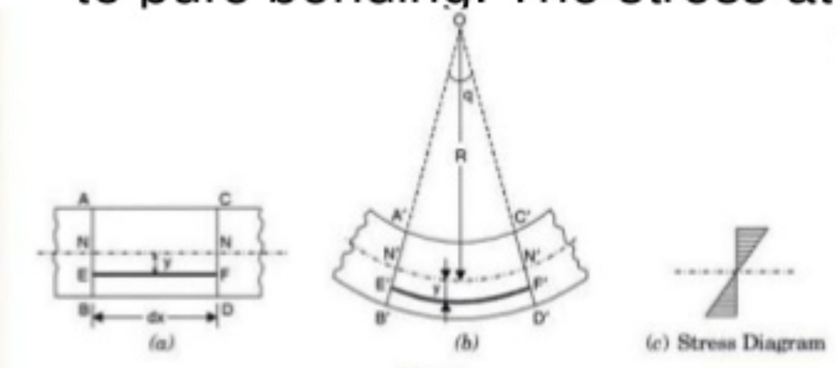


(c) Stress Diagram

4.5 NEUTRAL AXIS

- For a beam subjected to a pure bending moment, the stresses generated on the neutral layer is zero.
- Neutral axis is the line of intersection of neutral layer with the transverse section
- Consider the cross section of a beam subjected to pure bending. The stress at a distance y from

=E/R



4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Bending equation is applicable to a beam subjected to pure/simple bending. Ie the bending moment acting on the beam is constant and the shear stress is zero
- However in practical applications, the bending moment varies from section to section and the shear force is not zero
- But in the section where bending moment is maximum, shear force (derivative of bending moment) is zero
- Hence the bending equation is valid for the section where bending moment is maximum

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Or in other words, the condition of simple bending may be satisfied at a section where bending moment is maximum.
- Therefore beams and structures are designed using bending equation considering the section of maximum bending moment
- Flexural rigidity/Flexural resistance of a beam:-
- For pure bending of uniform sections, beam will deflect into circular arcs and for this reason the term circular bending is often used.